Models to Estimate Longitudinal Compressive Strength of Brazilian Hardwood Based on Apparent Density

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As wood is an orthotropic and natural material, there are several properties required for its use in civil construction. The apparent density has been used to estimate physical and mechanical properties of wood, as it is easy to determine experimentally, unlike other determinations, which involve the use of equipment available only in large research centers. Using the Brazilian standard ABNT NBR 7190 and linear and non-linear regression models, this research aimed to evaluate their accuracy in estimating the compressive strength parallel to the fibers ($f_{c0}$) as well as their characteristic value ($f_{c0,k}$). This study considered 72 tropical wood species from native forests that were divided into the 4 strength classes of this standard. For the set formed by all species, the linear polynomial model was the best fit, resulting in a determination coefficient of just over 70%.

Keywords: Hardwoods; Longitudinal compressive strength; Apparent density; Regression models

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INTRODUCTION

Wood is a material with excellent structural properties, mainly because it has strength in traction and compression. Another important point is that it is a renewable resource, unlike steel and concrete, which are other materials commonly used in construction (Pigozzo et al. 2018a). According to Bacha (2017), Brazil stands out on the international scene for its extensive tropical native forests and for planting homogeneous forests with exotic species. Also, Brazil has 493.5 million hectares of forests, which is equivalent to 12% of the world's forests.

The Brazilian standard ABNT NBR 7190 (1997), which establishes the premises and methods for wooden structures design, defines four strength classes for the categorization of any hardwood species for structural purposes. This classification is based on the characteristic compressive strength value in the direction parallel to the fibers ($f_{c0,k}$), as shown in Table 1, where $E_{c0,m}$ consists of the mean value of the elasticity module to compression in the direction parallel to the fibers and $\rho_{12\%}$ is the apparent density relative to the equilibrium moisture content (12%) considered by the standard.
Table 1. Strength Classes of Hardwoods According to ABNT NBR 7190 (1997)

<table>
<thead>
<tr>
<th>Classes</th>
<th>Range</th>
<th>$E_{c0,m}$ (MPa)</th>
<th>$\rho_{12%}$ (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C20</td>
<td>$20 \leq f_{c0,k} &lt; 30$ MPa</td>
<td>9500</td>
<td>650</td>
</tr>
<tr>
<td>C30</td>
<td>$30 \leq f_{c0,k} &lt; 40$ MPa</td>
<td>14500</td>
<td>800</td>
</tr>
<tr>
<td>C40</td>
<td>$40 \leq f_{c0,k} &lt; 60$ MPa</td>
<td>19500</td>
<td>950</td>
</tr>
<tr>
<td>C60</td>
<td>$f_{c0,k} \geq 60$</td>
<td>24500</td>
<td>1000</td>
</tr>
</tbody>
</table>

For designing, it is ideal to know all the material properties of strength and stiffness to be used (Oliveira et al. 2019). For wood, an orthotropic material, several tests are necessary to obtain the properties of interest, some of which are performed using specialized equipment, available only in large research centers. In addition to the simplified method contained in ABNT NBR 7190 (1997), an alternative process that establishes relationships between characteristic values of strength to certain stresses consists of estimating the properties (physical or mechanical) using apparent density as a dependent variable (Adamopoulos and Passialis 2010; Pigozzo et al. 2018b).

The apparent density is a property that is easily determined experimentally, defined by the ratio between the mass and the volume of the sample at 12% humidity, as indicated in the 1997 standard, ABNT NBR 7190 (Pigozzo et al. 2018c). Thus, the estimation of other wood properties with just one simple test is extremely valuable (Almeida et al. 2019; Wolenski et al. 2020; Almeida et al. 2020), allowing the engineer to carry out structural design easily and effectively (Lahr et al. 2016).

There are several reports using apparent density as an estimator of physical and mechanical properties of wood from native Brazilian forests (Lahr et al. 2016; Almeida et al. 2017; Christoforo et al. 2017). However, few studies that have considered the number of species (72) evaluated in the present research, which makes it possible to effectively verify the accuracy and generality of the use of apparent density, via regression models, in the estimation of the strength to parallel compression ($f_{c0}$) and its associated characteristic value ($f_{c0,k}$). It is also noteworthy that, depending on the precision of the models to be obtained, they can be used in the pre-classification of the wood right after its cut (pre-classification in loco) and naturally in the design of structures by incorporating such results (models) in future versions of the Brazilian standard ABNT NBR 7190 (1997).

With that in mind, this research aimed to estimate, using the linear and nonlinear regression models and the Brazilian standard ABNT NBR 7190 (1997), the precision in the estimation of the compressive strength parallel to the fibers ($f_{c0}$) and the characteristic value ($f_{c0,k}$) by means of the apparent density considering 72 species of hardwood from native Brazilian forests.

**EXPERIMENTAL**

**Materials**

The 72 species considered in the present study are shown in Table 2. The wood from homogeneous lots was properly stored in a greenhouse, resulting in a moisture content close to 12% in accordance with ABNT NBR 7190 (1997). Twelve specimens per species were manufactured and tested in parallel compression to the fibers (Fig. 1), and another twelve specimens were used to determine the apparent density values, resulting in 1728 experiments.
Table 2. Wood Species Evaluated

<table>
<thead>
<tr>
<th>Common Name</th>
<th>RC</th>
<th>Common Name</th>
<th>RC</th>
<th>Common Name</th>
<th>RC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amescla</td>
<td>C20</td>
<td>Canela Parda</td>
<td>C40</td>
<td>Peroba Mica</td>
<td>C60</td>
</tr>
<tr>
<td>Caixeta</td>
<td>C20</td>
<td>Canelão</td>
<td>C40</td>
<td>Umirana</td>
<td>C60</td>
</tr>
<tr>
<td>Cajueiro</td>
<td>C20</td>
<td>Casca Grossa</td>
<td>C40</td>
<td>Angelim Ferro</td>
<td>C60</td>
</tr>
<tr>
<td>Cambará</td>
<td>C20</td>
<td>Copaiba</td>
<td>C40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cambará Rosa</td>
<td>C20</td>
<td>Cupuíba</td>
<td>C40</td>
<td>Angelim Vermelho</td>
<td>C60</td>
</tr>
<tr>
<td>Cedro Doce</td>
<td>C20</td>
<td>Goiabão</td>
<td>C40</td>
<td>Angico Preto</td>
<td>C60</td>
</tr>
<tr>
<td>Cedro</td>
<td>C20</td>
<td>Louro Verde</td>
<td>C40</td>
<td>Breu Vermelho</td>
<td>C60</td>
</tr>
<tr>
<td>Cedroarana</td>
<td>C20</td>
<td>Mirarema</td>
<td>C40</td>
<td>Champanhe</td>
<td>C60</td>
</tr>
<tr>
<td>Marupá</td>
<td>C20</td>
<td>Piolho</td>
<td>C40</td>
<td>Cutiúba</td>
<td>C60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Quaruva Rosinha</td>
<td>C40</td>
<td>Garrote</td>
<td>C60</td>
</tr>
<tr>
<td>Quarubarana</td>
<td>C20</td>
<td>Rabo de Arraia</td>
<td>C40</td>
<td>Guajarã</td>
<td>C60</td>
</tr>
<tr>
<td>Cambará</td>
<td>C30</td>
<td>Angelim Pedra</td>
<td>C60</td>
<td>Ipé</td>
<td>C60</td>
</tr>
<tr>
<td>Castanheira</td>
<td>C30</td>
<td></td>
<td></td>
<td>Itaúba</td>
<td>C60</td>
</tr>
<tr>
<td>Cedro Amazonense</td>
<td>C30</td>
<td>Angelim Saia</td>
<td>C60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cedro</td>
<td>C30</td>
<td>Casca Grossa</td>
<td>C60</td>
<td>Jatobá</td>
<td>C60</td>
</tr>
<tr>
<td>Cupuíba</td>
<td>C30</td>
<td>Castelo</td>
<td>C60</td>
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<td></td>
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<tr>
<td>Embireira</td>
<td>C30</td>
<td>Catanudo</td>
<td>C60</td>
<td>Maçaranduba</td>
<td>C60</td>
</tr>
<tr>
<td>Tauari</td>
<td>C30</td>
<td>Envira Branca</td>
<td>C60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Abiú</td>
<td>C40</td>
<td>Garapa</td>
<td>C60</td>
<td>Oiuchu</td>
<td>C60</td>
</tr>
<tr>
<td>Angelim Amargoso</td>
<td>C40</td>
<td>Guaicara</td>
<td>C60</td>
<td>Quina Rosa</td>
<td>C60</td>
</tr>
<tr>
<td>Angelim Araroba</td>
<td>C40</td>
<td>Guanandi</td>
<td>C60</td>
<td>Roxinho</td>
<td>C60</td>
</tr>
<tr>
<td>Angico Branco</td>
<td>C40</td>
<td>Guarucia</td>
<td>C60</td>
<td>Sucupira</td>
<td>C60</td>
</tr>
<tr>
<td>Bicuíba</td>
<td>C40</td>
<td>Ipé</td>
<td>C60</td>
<td>Tachi</td>
<td>C60</td>
</tr>
<tr>
<td>Branquillo</td>
<td>C40</td>
<td>Louro Preto</td>
<td>C60</td>
<td>Tatajuba</td>
<td>C60</td>
</tr>
<tr>
<td>Cafearana</td>
<td>C40</td>
<td>Oiticica Amarela</td>
<td>C60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Canafistula</td>
<td>C40</td>
<td>Parinari</td>
<td>C60</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1. a) Standardized specimen illustration (5 x 5 x 15 cm — ABNT NBR 7190, 1997); and b) the apparatus used to perform the compression tests parallel to the grain.

The specimens were tested on the universal testing machine (AMSLER, 25-ton load capacity, São Carlos, Brazil). Their moisture content (U) at the test time was obtained using the Marrari M5 contact humidity meter (10.22% ≤ U ≤ 12.73%; São Carlos, Brazil).

Methods – Characteristic Strength

With the sample moisture content obtained, the values of strength to parallel compression ($f_{c0}$) and consequently the characteristic values ($f_{c0,k}$) were corrected to a
moisture content of 12% (Eq. 1), in which \( f_{c0,U} \) is the strength of the sample associated with the moisture content \( U \) and \( f_{c0,12} \) is the strength with 12% moisture.

\[
f_{c0,12} = f_{c0,U} \left[ 1 + \frac{2 \cdot (U - 12)}{100} \right]
\]

(1)

Based on the corrected values of compressive strength in the direction parallel to the fibers (\( f_{c0,12} \)), Eq. 2 was used to determine the characteristic value (\( f_{c0,k} \)) for wood categorization, where \( f_1, f_2, \ldots, f_n \) denote the compressive strength values in ascending order of the 12 (n) specimens tested (ABNT NBR 7190 1997).

\[
f_{c0,k} = \text{Max} \left\{ \frac{f_1}{1.1 \cdot 2 \left( \frac{f_1 + f_2 + f_3 + \ldots + f_{(n/2)-1}}{(n/2)-1} \right) - f_{n/2}} \right\}
\]

(2)

**Methods – Regression Models**

In Eqs. 3 to 7, linear and nonlinear regression models used to estimate the values of \( f_{c0} \) and \( f_{c0,k} \) as a function of apparent density (\( \rho_{12\%} \)) are presented, in which \( \beta_0, \beta_1, \) and \( \beta_2 \) consist of the parameters to be adjusted by the least squares method.

\[
f_{c0}(\rho_{12\%}) = \beta_0 \cdot \rho_{12\%} + \beta_1 \]

(3)

\[
f_{c0}(\rho_{12\%}) = \beta_0 \cdot \ln(\rho_{12\%} + \beta_1) + \beta_2 \]

(4)

\[
f_{c0}(\rho_{12\%}) = \beta_0 \cdot \rho_{12\%}^{-\beta_1} + \beta_2 \]

(5)

\[
f_{c0}(\rho_{12\%}) = \left( \frac{\beta_0 + \beta_1}{\rho_{12\%} + \beta_2} \right) \]

(6)

\[
f_{c0}(\rho_{12\%}) = (\beta_0 + \beta_1 \cdot \rho_{12\%})^2 \]

(7)

For \( f_{c0} \), regression models were generated for each of the four strength classes (Table 1) and also for the set involving the 72 species. For \( f_{c0,k} \) a model was obtained considering only the set of all species. It should be noted that \( f_{c0,k} \) is the property that categorizes a given species of wood in one of the strength classes of the Brazilian standard. Predictive models were evaluated using the coefficient of variation (CV – Eq. 8), the average absolute percentage error (MAPE – Eq. 9), and the determination coefficient (R² - Eq. 10).

\[
CV(\%) = 100 - \sqrt{\frac{\sum_{i=1}^{n} (Y_{predict,i} - Y_{data,i})^2}{\sum_{i=1}^{n} Y_{data,i}^2}}
\]

(8)

\[
\text{MAPE}(\%) = 100 \cdot \frac{1}{n} \sum_{i=1}^{n} \left| \frac{Y_{predict,i} - Y_{data,i}}{Y_{data,i}} \right|
\]

(9)
\[ R^2(\%) = 100 \cdot \left( 1 - \frac{\sum_{i=1}^{n} (Y_{\text{predict}}_i - Y_{\text{data}}_i)^2}{\sum_{i=1}^{n} (Y_{\text{data}}_i - \bar{Y}_{\text{data}})^2} \right) \] (10)

From Eqs. 8 to 9, \( n \) is the considered sample’s number, \( Y_{\text{predict}}_i \) (\( f_{c0} \) or \( f_{c0,k} \)) is the value estimated by the regression model, \( Y_{\text{data}}_i \) is the experimentally determined value, and \( \bar{Y}_{\text{data}} \) is the property average value obtained experimentally.

**RESULTS AND DISCUSSION**

**Physical and Mechanical Properties**

Figure 2 shows the results of apparent density, compression strength parallel to fibers, and the characteristic value of strength to parallel compression, with CV being the variation coefficient. The CV is presented showing the lowest and highest values obtained in all studied species (20 to 60), proving the quality of the tests performed. The bars represent the confidence intervals of the averages (the average varies from the lowest to the highest value shown in the bars), using 95% reliability.

![Graphs showing physical and mechanical properties](image)

**Fig. 2.** Results obtained in the tests performed: a) bulk density; b) compression parallel to the fibers; c) shear parallel to the fibers; and d) species frequency by strength class

From Fig. 2, the average values of apparent density by strength classes were all lower than the reference values presented in Table 1. It should be noted that after almost 23 years, the species number considered in this research is surely higher than the species number that gave rise to the results in Table 1. There is a trend of linear relationship when the values of \( f_{00} \) with \( \rho_{12\%} \), and \( f_{00,k} \) with \( \rho_{12\%} \), are observed.

The compressive strength results clearly increase progressively with the increase in the strength class, evidencing that the variation coefficient was less than 18%, as required by ABNT NBR 7190 (1997) for normal stresses. Of the characteristic values, 13.92% of the species were classified in class C20, 10.13% were class C30, 24.10% were class C40, and 51.85% in class C60.

### Regression Models

Table 3 presents the best adjustments (considering Eqs. 3 to 7) by strength class for the estimate of \( f_{00} \) as well as the models of \( f_{00} \) and \( f_{00,k} \) considering the results for the 72 wood species.

#### Table 3. Better Adjustments by Strength Class and for the Set Considering All Wood Species

<table>
<thead>
<tr>
<th>SC</th>
<th>Regression Adjustment</th>
<th>CV (%)</th>
<th>MAPE (%)</th>
<th>( R^2 ) (%)</th>
<th>( R^2 ) adj</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>( f_{00}(\rho_{12%}) = -41.32 + 3.39 \ln (\rho_{12%} - 742.13) )</td>
<td>10.73</td>
<td>9.81</td>
<td>19.97</td>
<td>15.35</td>
</tr>
<tr>
<td>30</td>
<td>( f_{00}(\rho_{12%}) = 1.96 \cdot \ln (\rho_{12%} - 496.20) + 36.51 )</td>
<td>2.26</td>
<td>2.33</td>
<td>83.76</td>
<td>74.87</td>
</tr>
<tr>
<td>40</td>
<td>( f_{00}(\rho_{12%}) = -62.33 \ln (\rho_{12%} - 1.2510^9) )</td>
<td>10.81</td>
<td>8.91</td>
<td>23.44</td>
<td>20.12</td>
</tr>
<tr>
<td>60</td>
<td>( f_{00}(\rho_{12%}) = 8.36 \ln (\rho_{12%} - 643.21) + 33.11 )</td>
<td>10.09</td>
<td>8.38</td>
<td>11.45</td>
<td>7.54</td>
</tr>
<tr>
<td>All Species</td>
<td>( f_{00}(\rho_{12%}) = 0.074 \cdot \rho_{12%} + 1.80 )</td>
<td>14.67</td>
<td>11.19</td>
<td>73.30</td>
<td>65.32</td>
</tr>
<tr>
<td>All Species</td>
<td>( f_{0,4}(\rho_{12%}) = 0.068 \cdot \rho_{12%} - 2.562 )</td>
<td>16.76</td>
<td>13.54</td>
<td>71.20</td>
<td>64.87</td>
</tr>
</tbody>
</table>

The regression models for the \( f_{00} \) estimate by strength classes, with the exception of C30, showed little precision (less than 23.44%). These results are due to the small variation in density within the strength classes, implying that it is not possible to capture a trend of behavior between \( \rho_{12\%} \) and \( f_{00} \) with the increase in apparent density values. Such a result can also be seen by the regression models (best adjustments) not being the same in all strength classes.

For class C30, which corresponds to 10.13% of the total evaluated species, the value of \( R^2 \) (83.76%) is indicative of a strong relationship between anatomy, apparent density, and strength to compression parallel to the fibers (Duarte et al. 2020). It should be noted that the same precision degree (\( R^2 = 19.97\% \)) was not obtained for class C20, even with a higher number of species (13.92% of the total) when compared to C30.

For the set involving all species, both for the estimate of \( f_{00} \) and \( f_{0,0,k} \), the linear model consisted of the best fit, resulting in determination coefficients close to 70%, which implies that the error made in these variables is around 30% (Montgomery 2013). As the species number is significant, the failure to obtain even more accurate models is an indication that other variables should be used in wood properties prediction, such as chemical constituents and anatomical parameters. However, it is noteworthy that the essence of this research was to verify the model’s accuracy considering only the apparent
density, as it is a property of easy experimental determination, thus collaborating to enable
the more rational use of wood in structural projects, results that should be considered in
new versions of the Brazilian standard ABNT NBR 7190 (1997).

The research presented by Christoforo et al. (2020) also evaluated the apparent
density as an estimator of other properties of 10 hardwood species. However, based on just
the density data it was not possible to estimate any properties of these woods. Unlike the
present study, the authors used only linear, exponential, logarithmic, and geometric
models, which may help to account for a bad fit. The same is confirmed in the work of
Almeida et al. (2017), in which these adjustments were not optimal to estimate most of the
properties analyzed. The apparent density was able to estimate only the basic density and
anhydrous density, obtaining a quality of 72.9% and 99.7%, respectively, using the
geometric model.

Figure 3 shows the adjustments obtained in the estimate of $f_{c0}$ and $f_{c0,k}$ considering
the set of all species. It is possible to note that the higher the density of the wood species,
the greater its compression strength. This may be due to denser structures having a higher
proportion of lignin.

![Fig. 3. a) regression model using the apparent density as an estimation of the compressive
strength parallel to the fibers; and b) regression model using apparent density as an estimation
of the compressive strength parallel to the characteristic fibers.](image)

**CONCLUSIONS**

1. The small variation in apparent density within a given strength class, with the
exception of C30, implies that regression models cannot be used effectively to estimate
of compressive strength in the direction parallel to the fibers ($f_{c0}$) within that class.

2. With all species considered, the linear regression model was the best fit, both for
estimating $f_{c0}$ and predicting its characteristic value ($f_{c0,k}$).

3. Because the determination coefficient was close to 70% (slightly higher), the linear
regression models obtained in the present study are presented as alternatives in the
compressive strength estimation as well as for the pre-classification of a given species.
These findings should be taken into consideration in future versions of the Brazilian
normative document ABNT NBR 7190 (1997).
REFERENCES CITED


